CHALLENGES OF VIX-SPX FUTURE SPREAD FORECASTING

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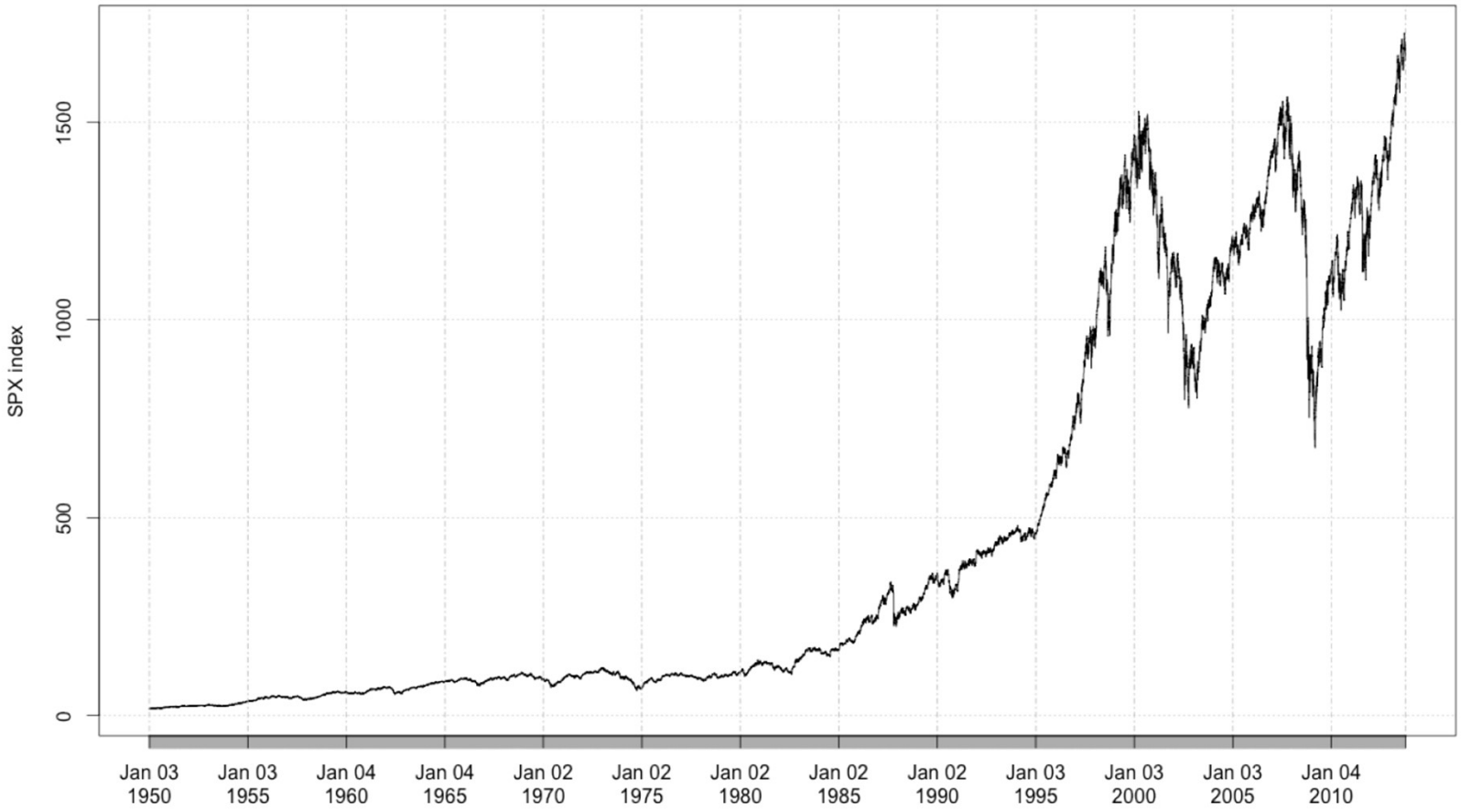
SPX INDEX

SPX is one of the tickers for the S&P 500 index.

From "S&P 500" Wikipedia: The Free Encyclopedia.

> *The S&P 500, or the Standard & Poor's 500, is a stock market index based on the market capitalizations of 500 large companies having common stock listed on the NYSE or NASDAQ. The S&P 500 index components and their weightings are determined by S&P Dow Jones Indices. It differs from other U.S. stock market indices such as the Dow Jones Industrial Average and the Nasdaq Composite due to its diverse constituency and weighting methodology. It is one of the most commonly followed equity indices and many consider it the best representation of the U.S. stock market as well as a bellwether for the U.S. economy.*

TIME SERIES OF SPX SINCE 1950

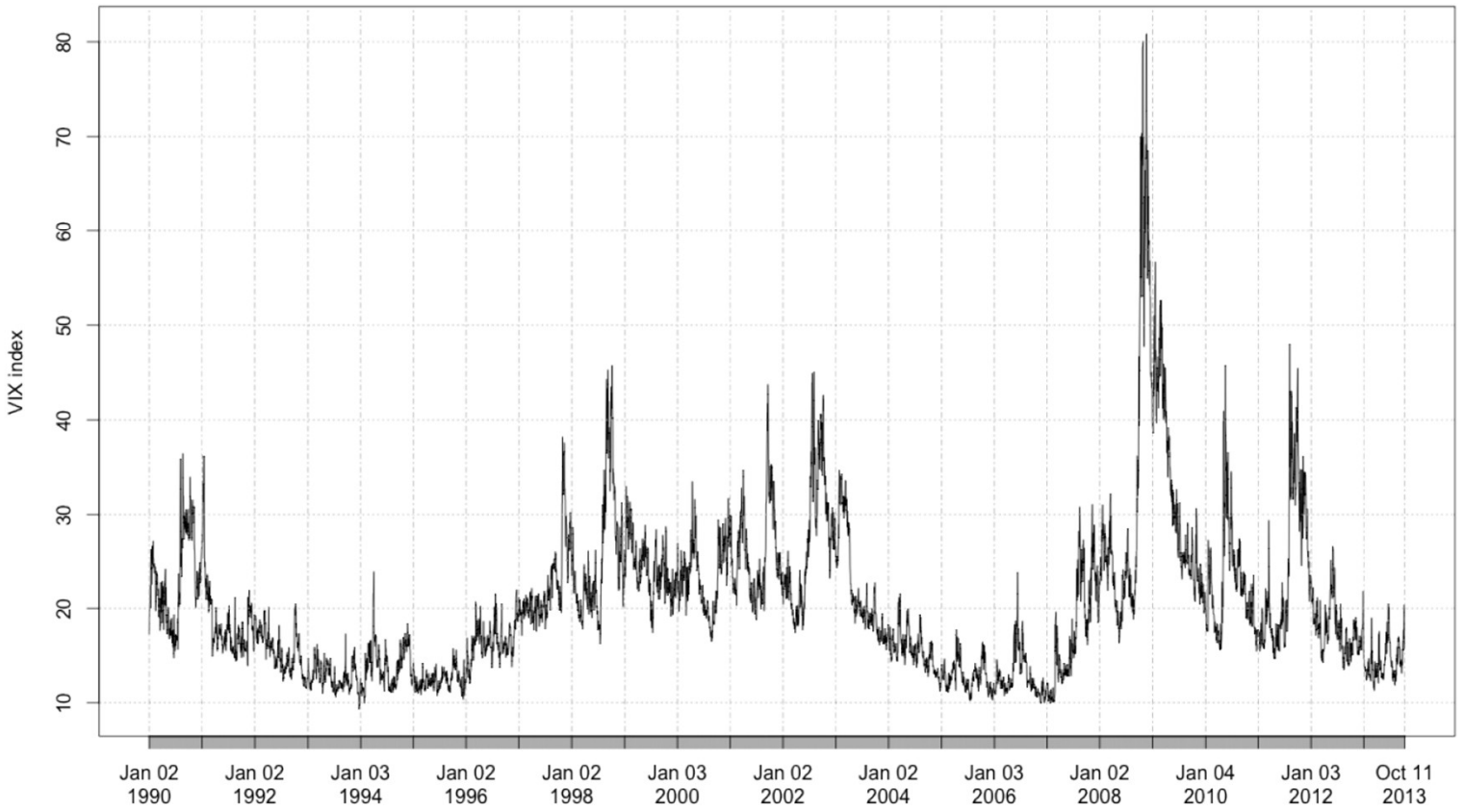


VIX INDEX

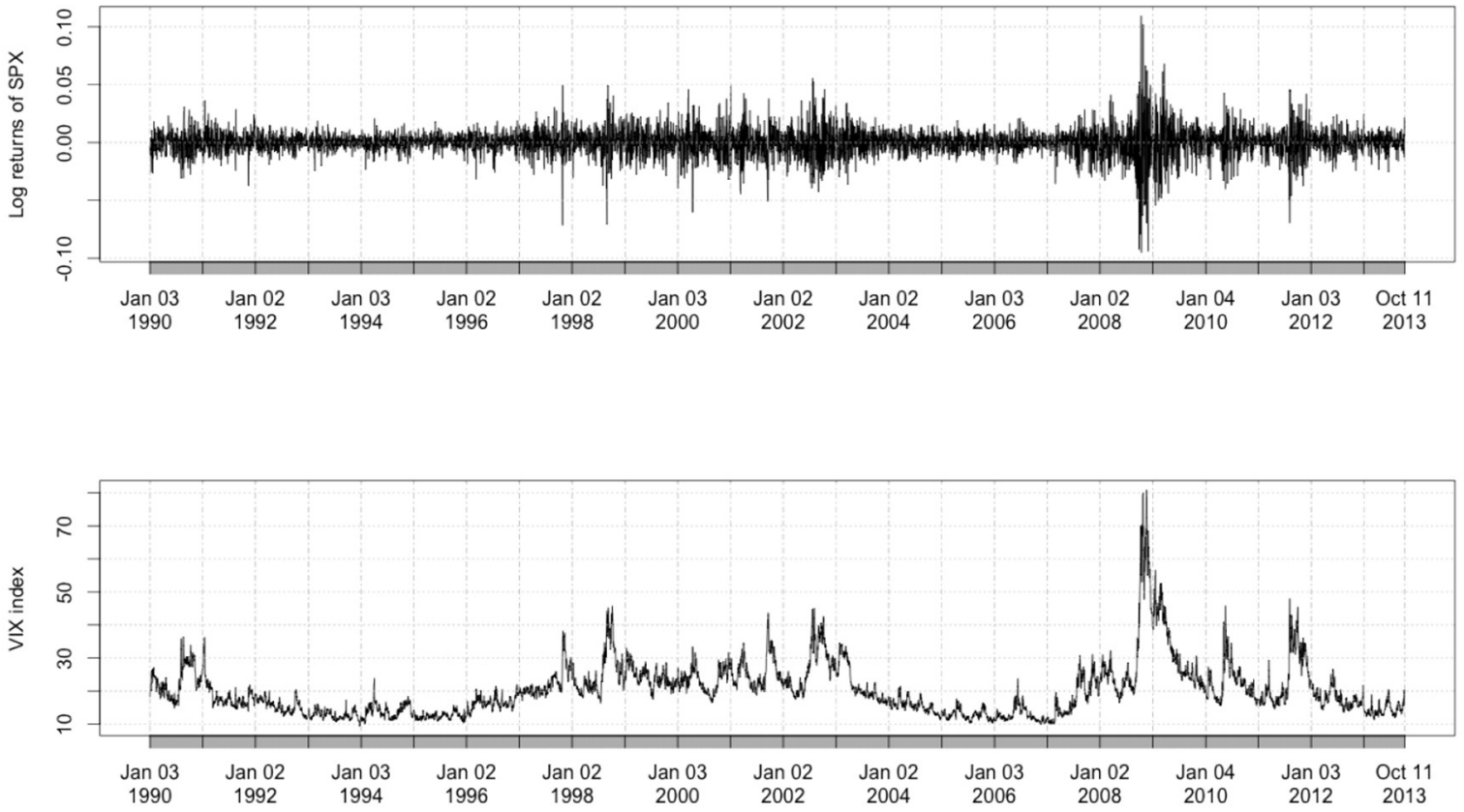
From "VIX" Wikipedia: The Free Encyclopedia.

> *VIX is a trademarked ticker symbol for the Chicago Board Options Exchange Market Volatility Index, a popular measure of the implied volatility of S&P 500 index options. Often referred to as the fear index or the fear gauge, it represents one measure of the market's expectation of stock market volatility over the next 30 day period.*

TIME SERIES OF VIX SINCE 1990



VIX IS A MEASURE OF VOLATILITY



OPTIONS

From "Option (finance)" Wikipedia: The Free Encyclopedia.

> *In finance, an option is a contract which gives the buyer (the owner) the right, but not the obligation, to buy or sell an underlying asset or instrument at a specified strike price on or before a specified date. The seller incurs a corresponding obligation to fulfill the transaction that is to sell or buy if the owner elects to "exercise" the option prior to expiration. The buyer pays a premium to the seller for this right. An option which conveys to the owner the right to buy something at a specific price is referred to as a call; an option which conveys the right of the owner to sell something at a specific price is referred to as a put. Both are commonly traded, but for clarity, the call option is more frequently discussed.*

OPTIONS ON SPX AND VIX

- In particular, there are options on SPX and options on VIX.
- We saw that the VIX index reflects the volatility of SPX.
- The values of options on SPX and options on VIX should be related.
- In the following, we will see some of the ways in which these option values are related.

VIX CALCULATION

The VIX measures expected volatility of S&P500 index in the next 30 days

$$\text{VIX} = \sigma \times 100, \qquad \sigma^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2$$

$Q(K_i)$	The midpoint of the bid-ask spread for each option with strike K_i.
K_i	Strike price of the ith out-of-the-money option; a call if $K_i > K_o$; and a put if $K_i < K_o$; both put and call if $K_i = K_o$.
T	Time to expiration
F	Forward index level desired from index option prices
K_o	First strike below the forward index level, F
R	Risk-free interest rate to expiration

OPTION VALUATION

- In mathematical finance, the value of an option is given by the expectation (under the risk neutral measure) of the final payoff conditional on the information available at the current time t.
- Specifically, for a European call option expiring at time T,

$$C(S, K, T) = \mathbb{E}\left[(S_T - K)^+ \middle| \mathcal{F}_t\right].$$

THE BLACK-SCHOLES MODEL

- Black and Scholes model the evolution of the underlying as

$$\frac{dS_t}{S_t} = \mu\, dt + \sigma\, dZ_t$$

with the volatility σ constant.

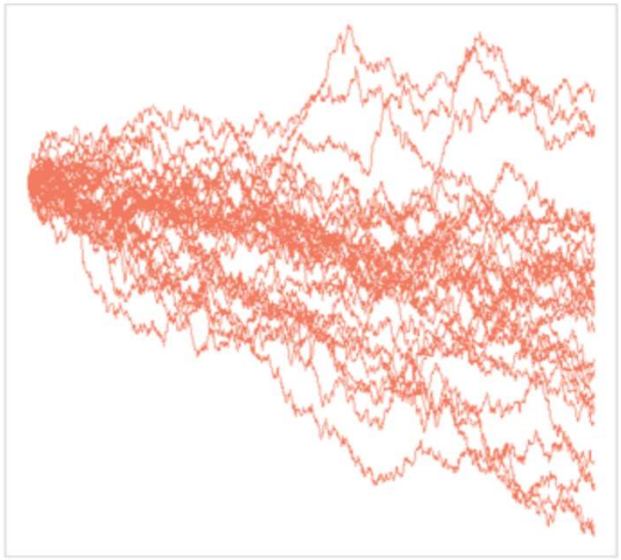
- The price of a European option is then given by the Black-Scholes formula:

$$C(S, K, T) = \mathbb{E}\left[(S_T - K)^+ \middle| \mathcal{F}_t\right] = PV\ \{F\,\mathcal{N}(d_1) - K\,\mathcal{N}(d_2)\}$$

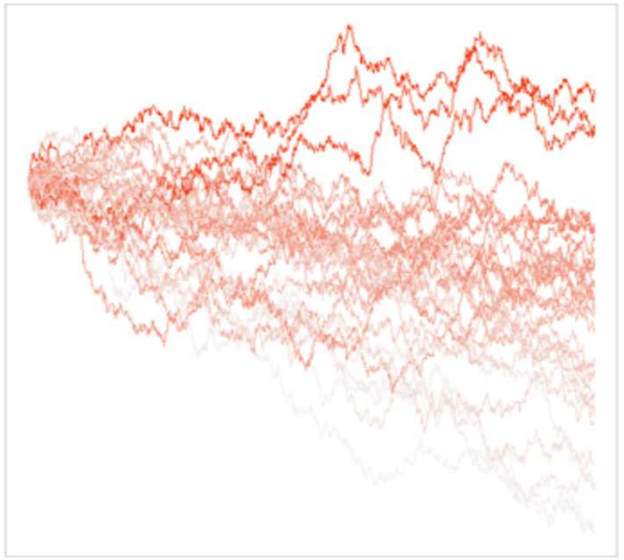
where F is the forward price, $\mathcal{N}(\cdot)$ is the cumulative normal distribution function and with $\tau = T - t$,

$$d_1 = \frac{\log F/K}{\sigma\sqrt{\tau}} + \frac{\sigma\sqrt{\tau}}{2}; \quad d_2 = \frac{\log F/K}{\sigma\sqrt{\tau}} - \frac{\sigma\sqrt{\tau}}{2}.$$

RISK-NEUTRAL VALUATION



30 paths of a Brownian motion with negative drift

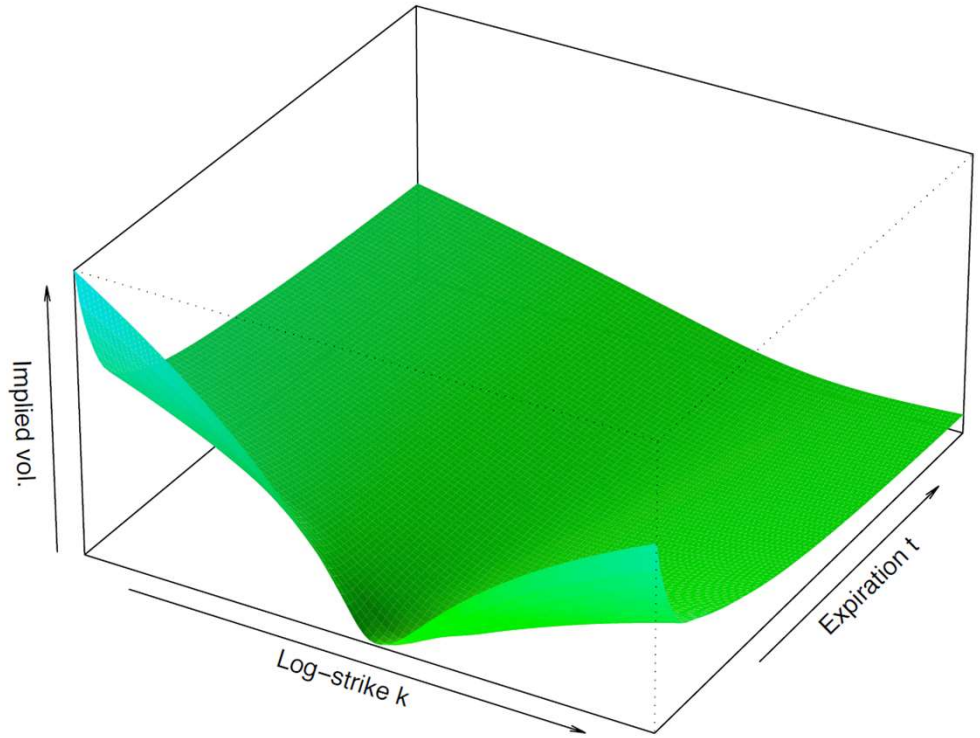


The same paths reweighted according to the Girsanov formula

Visualisation of the Girsanov theorem — The left side shows a Wiener process with negative drift under a canonical measure *P*; on the right side each path of the process is colored according to its likelihood under the martingale measure *Q*. The density transformation from *P* to *Q* is given by the Girsanov theorem.

IMPLIED VOLATILITY SURFACE FOR SPX

Here's a 3D plot of the volatility surface as of September 15, 2005:



$k := \log K/F$ is the log-strike and t is time to expiry.

VIX AS A VARIANCE SWAP

The necessary assumptions are:

- existence of futures market with delivery dates $T' \geq T$
- futures contract F_t(underlying) follows a diffusion process with no jumps
- existence of European futures options market, for these options all strikes are available (market is complete)
- continuous trading is possible

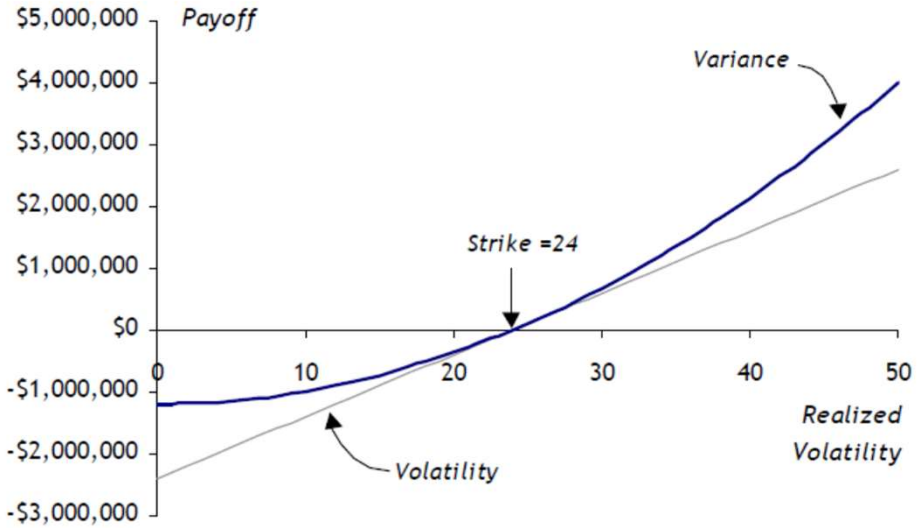
$$\frac{1}{T}\int_0^T \sigma_t^2 dt = \frac{2}{T}\left[\log\frac{F_0}{F_T} + \frac{F_T}{F_0} - 1\right] - \frac{2}{T}\int_0^T \left[\frac{1}{F_0} - \frac{1}{F_t}\right] dF_t$$

$$= \frac{2}{T}e^{rT}\int_0^{F_0} \frac{1}{K^2}P_0(K)dK + \frac{2}{T}e^{rT}\int_{F_0}^{\infty} \frac{1}{K^2}C_0(K)dK$$

VARIANCE VS VOLATILITY SWAPS

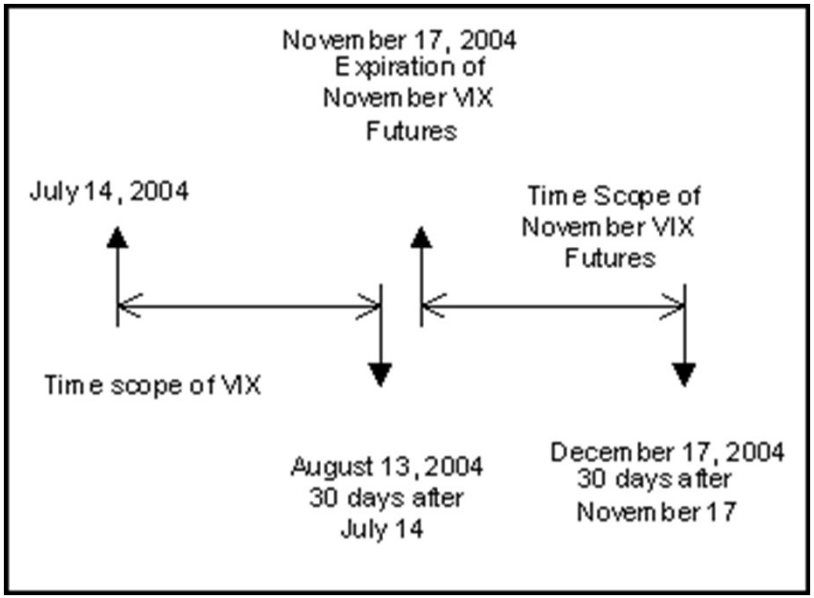
Jensen's inequality: $E(\sqrt{Variance}) \leq \sqrt{E(Variance)}$.

- The PnL from an options position is driven by realized variance, not volatility
- Variance swaps can be replicated using a static portfolio of European vanilla options, along with an equity position
- Variance swaps are more popular than volatility swaps - for which there exist only approximate static replication strategies

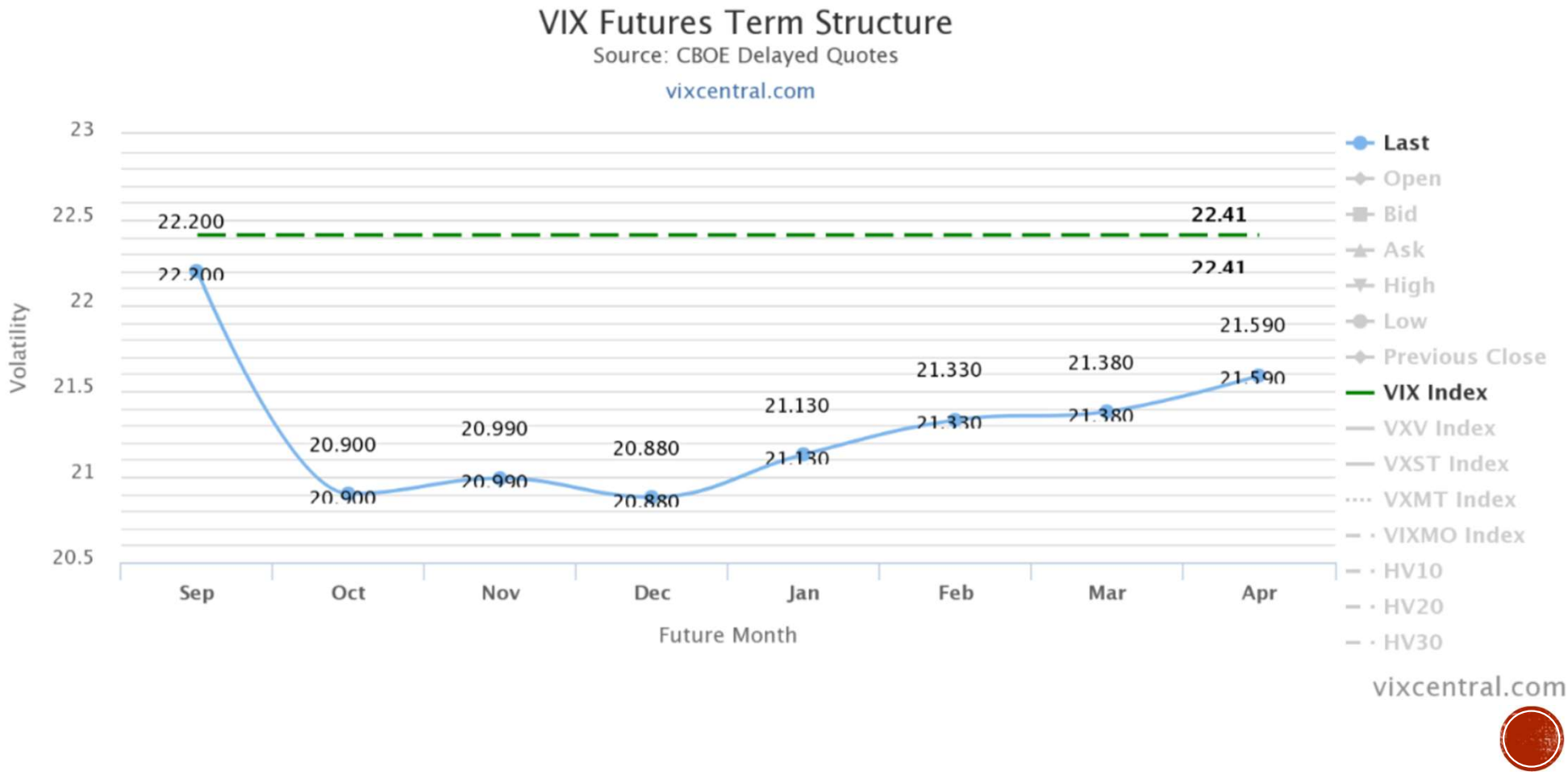


VIX FUTURES (VX)

- VIX futures is a forward contract on expected 30 day forward SPX total volatility



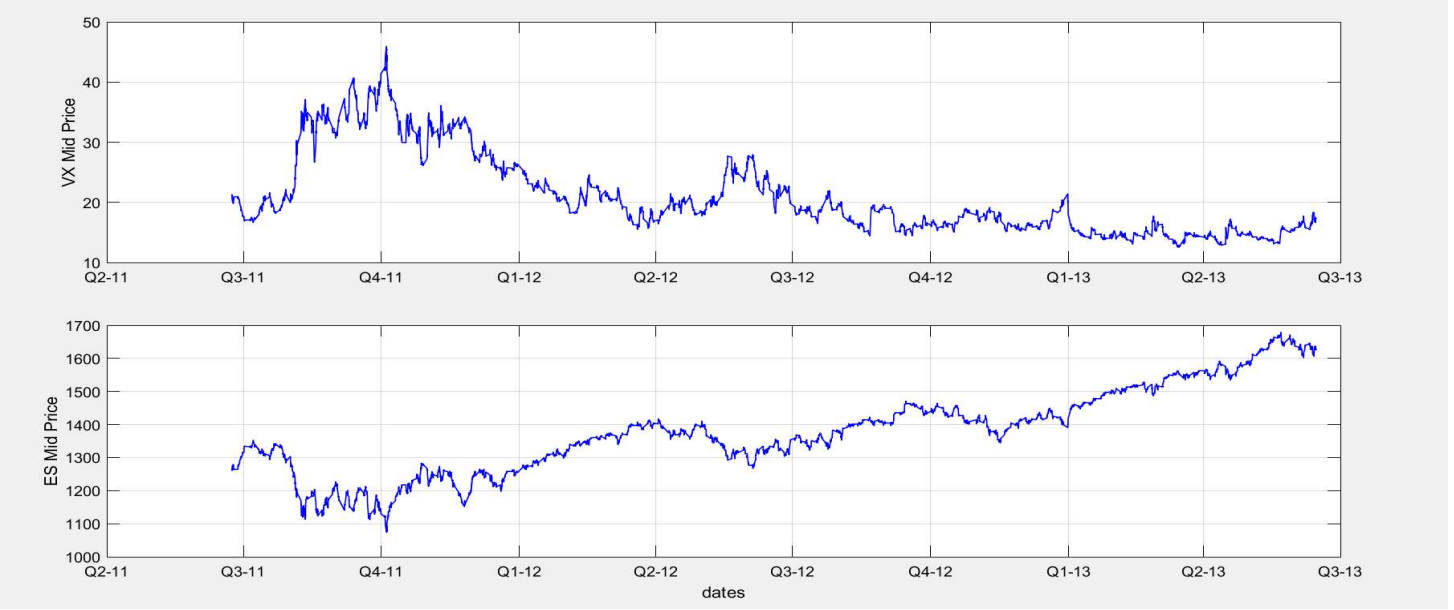
VIX FUTURES CONTANGO AND BACKWARDATION



VIX FUTURES BEHAVIOR

- In general as the SPX bull market looks to be pausing for a while do to global economic slowdown, if there is a lack of downside fear, then the VIX futures will remain in contango.

- When the market pulls back to recent lows and goes through those lows, then the VIX levels rise quickly and the implied vol reacts quickly causing backwardation and a large parallel shift causing the spread to rally quickly



VIX FUTURE CONVEXITY ADJUSTMENT AND VVIX

$$E(X^2) - (E(X))^2 = \text{var}(X)$$

$$(Fwd\ Var)^2 - (VIX\ Fut)^2 = \text{var}(VIX\ Fut)$$

$$\text{var}(VIX\ Fut) \approx (VIX\ Fut)^2 \times \frac{T}{365}\left(\frac{VVIX_T}{100}\right)^2$$

$VVIX_T$ refers to a $VVIX$ calculation with expiry T days in the future, not the standard 30 day calculation

The VVIX Index is an indicator of the expected volatility of the 30-day forward price of the VIX.

VIX AND SPX FUTURES AND OPTIONS

- **VIX options** – information about expected volatility of expected forward variance of SPX
- **SPX options** - expected risk-neutral distribution of SPX at expiration
- **VIX** - expected total variance of SPX

- **VIX options and futures** expiration: 3d Thursday of each month weekly 30 prior to SPX options expiration + weekly (starting 23-Jul-2015)

- **SPX options** expiration: 3d Friday of each month + weekly (SPXW) + end of month

- **SPX futures** expiration : quarterly, March, June, September and December

SVI PARAMETERIZATION OF IV SURFACE

Volatility information from options is hard to extract without a proper interpolation schema. This is where SVI parameterization by Jim Gatheral comes in.

$$\sigma_{BS}^2(k) = a + b\left\{\rho\,(k-m) + \sqrt{(k-m)^2+\sigma^2}\right\}$$

where the coefficients a, b, ρ, σ, and m depend on the expiration.

- It is relatively easy to fit listed option prices whilst ensuring no calendar spread arbitrage.
- For a fixed time to expiry t, the implied Black-Scholes variance is linear in log-strike for large values of k

IV VS LOCAL VOL. VS STOCHASTIC MODELS: PART 1

- IV is the easiest to work with but doesn't automatically provide an underlying diffusion process. But local vol. can be extracted from IV using

$$w(S_0, K, T) := \sigma_{BS}^2(S_0, K, T)\ T \qquad v_L = \frac{\frac{\partial w}{\partial T}}{1 - \frac{y}{w}\frac{\partial w}{\partial y} + \frac{1}{4}\left(-\frac{1}{4} - \frac{1}{w} + \frac{y^2}{w^2}\right)\left(\frac{\partial w}{\partial y}\right)^2 + \frac{1}{2}\frac{\partial^2 w}{\partial y^2}}$$

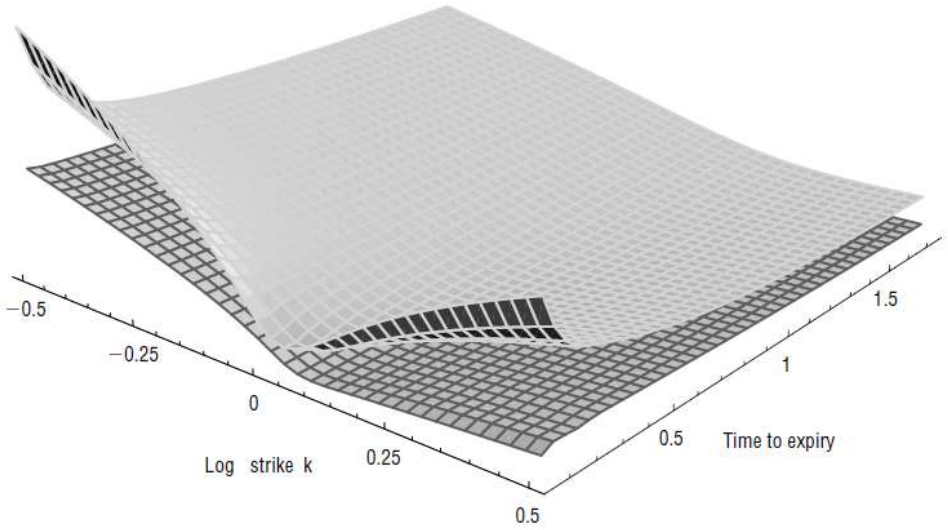
$$y = \log\left(\frac{K}{F_T}\right)$$

- Local vol requires calculating a second derivative of option price:

$$\sigma^2(K, T, S_0) = \frac{\frac{\partial C}{\partial T}}{\frac{1}{2}K^2\frac{\partial^2 C}{\partial K^2}}$$

IV VS LOCAL VOL. VS STOCHASTIC MODELS: PART 2

- All stochastic volatility models generate roughly the same shape of volatility surface and cannot fit option prices as shown below (empirical SPX IV vs Heston-based model on September 15, 2005)



JOINT FORECASTING OF IV SURFACES

- Inputs:
  - IV surface(s) historical dynamics
  - Underlying historical dynamics
  - Historical dynamics of various factors

- Components:
  - Factorization of IV surface movements
  - ML model for describing a joint behavior of factors
  - A separate model for IV surface transformation due to underlying moves

- Output:
  - Empirical joint distribution of IV surfaces conditional upon factors and underlying prices for a set of horizons

VIX/SPX FUTURE STRATEGY: TOOLS USED

- Execution System:

Trading Technologies API via Matlab Trading Toolbox

Trading Technologies Off-the-Shelf Tools

- Strategy Engine:

Matlab source compiled to C via Matlab Compiler Toolbox

- Backtesting Engine:

Matlab with Parallel Computing Toolbox

- Products Traded:

VIX Futures on CFE (VX)

S&P500 Mini Futures on CME Globex(ES)

VIX/SPX FUTURE STRATEGY: SOURCES OF PROFIT

The key to the strategy is the ability to trade in different market types with varying holding periods.

In general we can break the correlation strategy into 2 types of trading. Shorter holding periods (from intra-day to up to 2 days) and longer holding periods (longer than 3 days).

- Shorter holding period provide opportunities to dynamically trade both the underlying SPX risks as well as the local correlation of the SPX and the VIX.
- Longer holding periods encompass, correlation trending, volatility mean reversion and skew opportunities. In both profit centers, regime identification, risk scaling and hedge ratio determination dominate the PnL behavior of the strategy.

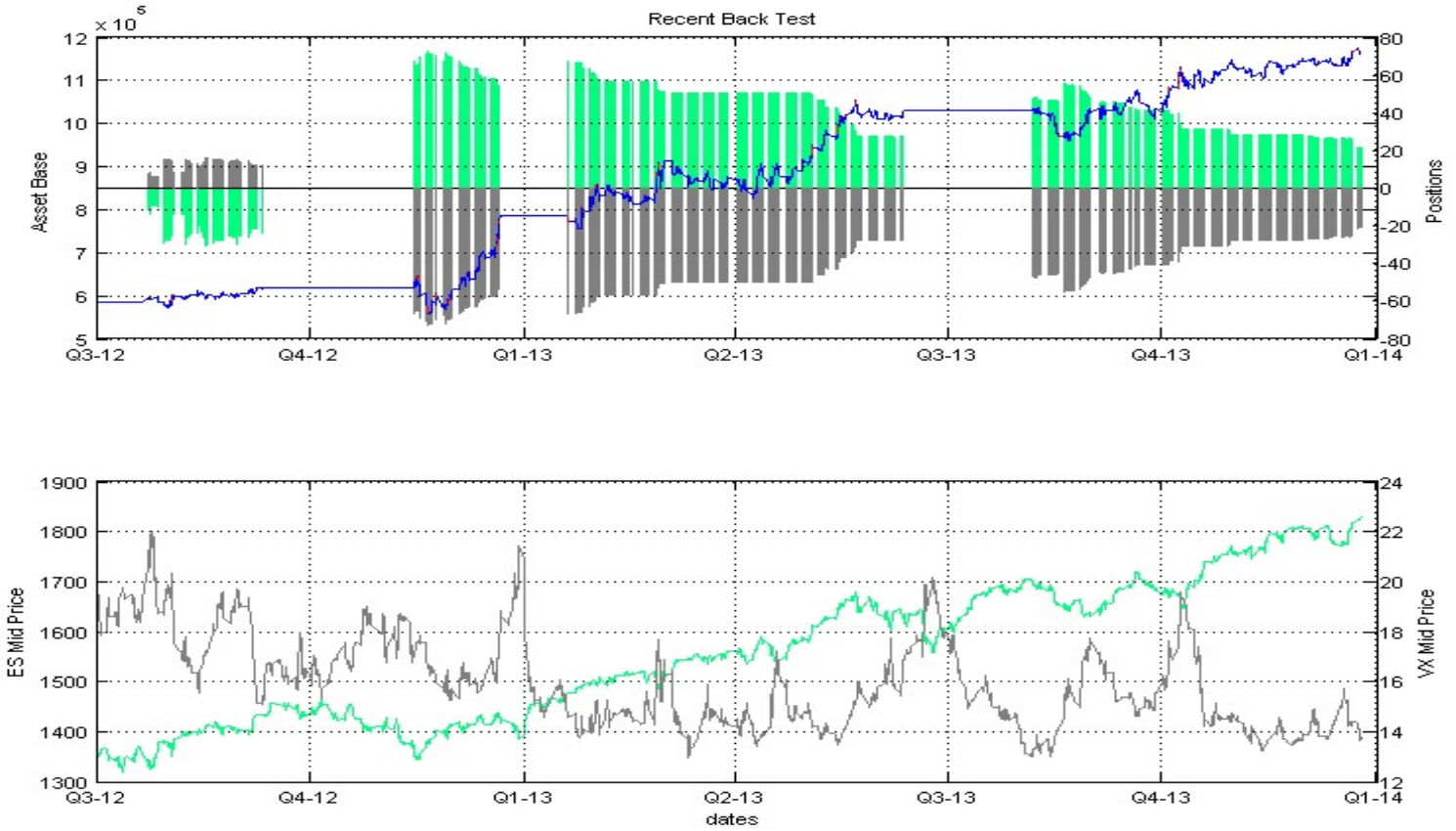
VIX/SPX FUTURE STRATEGY: SOURCES OF PROFIT: CONTINUED

- When new information arrives which could be material to the broad market direction, both the SPX and VIX react accordingly. Based on the type of information, broad market positive or broad market negative, both indices alter their local price movement behaviors and transition to new regimes. As this happens, the joint behavior of the two indices reacts as well. Understanding the impact of these new information arrivals creates longer term trading opportunities.

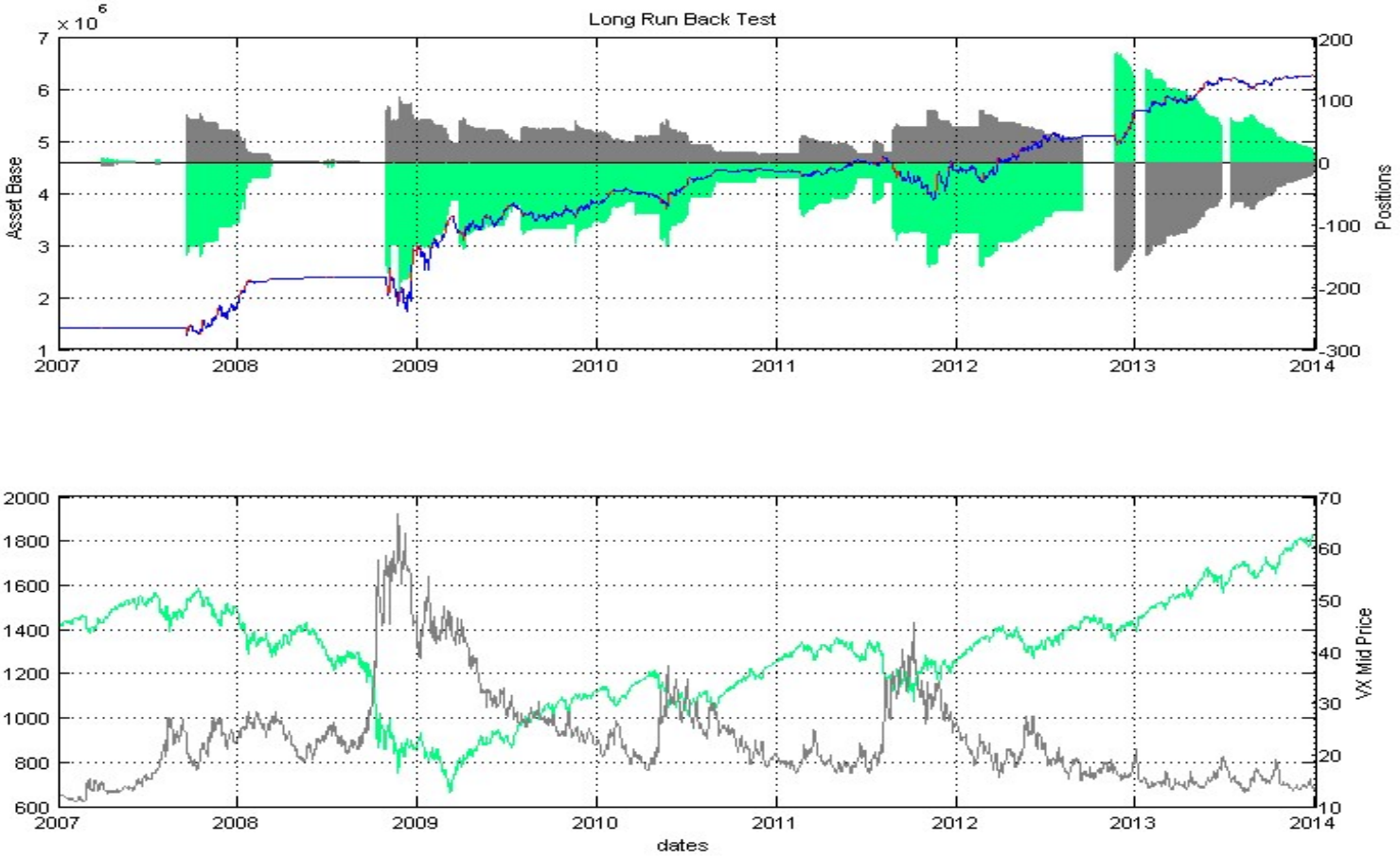
- In ES_Price+hedgeRatio*VX_Price hedge ratio is a strategy variable

- We trade multiple hedge ratios in parallel and switch between them depending on the strategy regime

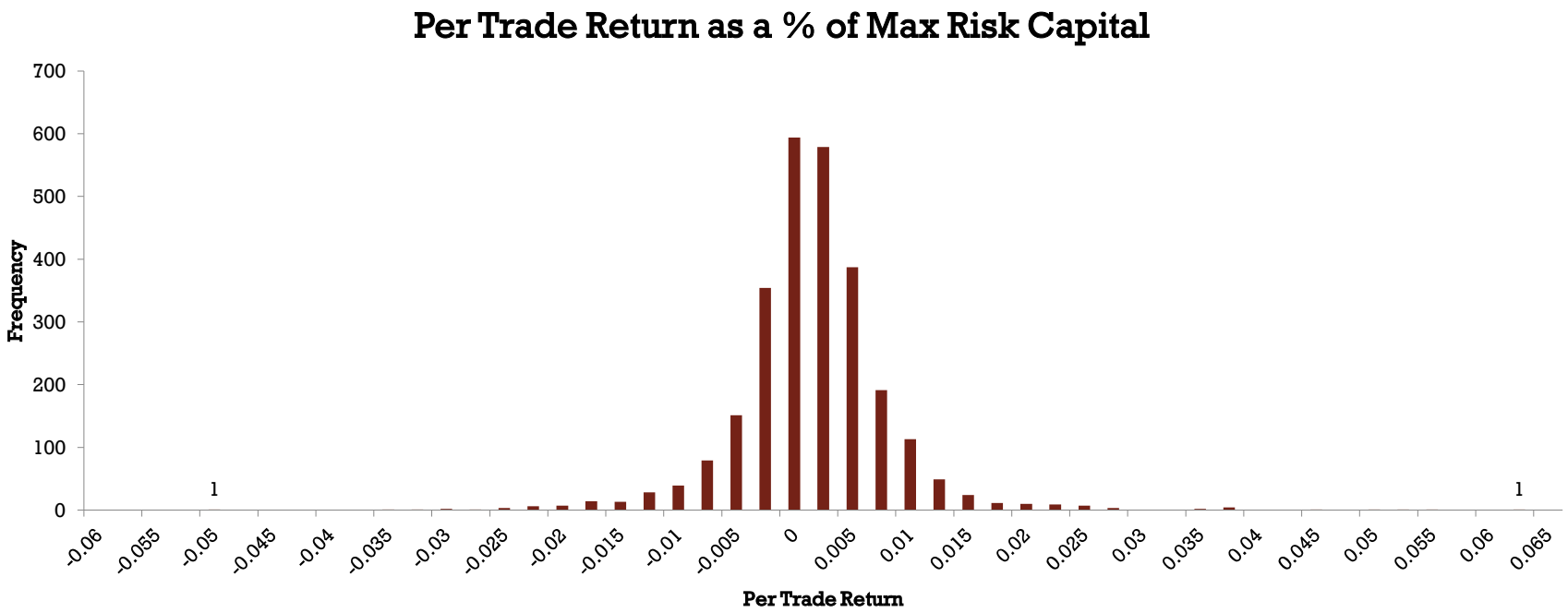
BACKTESTING RESULTS UP TO 2014



LONG RUN BACK TEST PNL PATH



BACK TEST RETURN HISTOGRAM



BACK TEST ASSUMPTIONS AND STATISTICS

- Last in queue fill assumption – full size must trade out to get order filled
- Full bid/ask price is paid on all non-passive trades
- No margin offset on hedges
- Conservative back testing results give 75% returns on less than $600K Risk Capital with limited drawdowns
- Period Jul2012-Dec2013
  - 450 trading Days, 2800 Trades
  - Ave Profit per trade $202
  - Max Risk Capital Amount less than $600K, Non-constrained
- Long Run Period Jan2007- Dec2013
  - 1730 trading Days, 5000 Trades
  - Ave Profit per trade $970
  - Max Risk Capital Amount less than $1.5MM, Non-constrained

TRADING IN 2015

- We made money in the back testing and live trading in low volatility regimes with some market movement.
- But, in very low realized volatility regimes, it is difficult to trade from the Long side with out a few instances of market movements because of the contango.
- Holding periods became quite long with very little "scalping" in and out of the spread to help pay for contango.
- Mark to market then caused losses in the PnL. However, like any long vol trade, when the market did move, implied vol responded very quickly, making the positions profitable again. We were not heavily funded and could not maintain our position mark to market and our development costs both at the same time.

IMPROVING STRATEGY FURTHER

- One solution to this type of very low volatility market is to trade from the short side, but we felt we needed better forecasts to do this.
- So we decided to do SPX option forecasting and VIX future forecasting (see next slide). We were almost complete with that when we lost finding.
- The other way to deal with the issue is to incorporate options into the strategy. We had small trades on for 2 months but no big gains as we could not back test options yet in our system.
- We were highly capital constrained so could only trade one product type. Including other products would highly increase an amount of trading opportunities.

VIX FUTURE FORECASTING

- For a fixed horizon use a separate model to build an empirical distribution of SPX for different horizons. This distribution works as a scenario i.e. WHAT IF input.
- Use joint IV surface forecasting model to predict an join distribution of (IV SPX,IV VIX) surfaces conditional upon SPX and VIX and other factors
- Build an probability distribution of SPX IV as a marginal distribution of joint distribution for (IV SPX, IV VIX)
- Use VIX formula and the marginal distribution to build VIX probability distribution
- Use the empirical distribution for SPX and the forecast from the previous step to build a joint distribution of (SPX,VIX)
- Use a separate regression model + convexity adjustment to build a joint distribution for (Frw SPX, Frw VIX) which allows us to bound a confidence channel for trading and decision making

STRATEGY IMPROVEMENT POTENTIAL

- The other products and market types can be handled well by the system as the vol of vol provides most of the system profits as can be seen by the older back test time periods.
- We can run a portfolio of pairs (spreads) like a multi-strategy trade to increase an amount of trading opportunities and lower risks.
- In general, all market types should show improved trading results as we finish the forecasts for the vix forwards and thus improved forecasts for the spreads.

BIBLIOGRAPHY

- [1] CBOE Volatility index – VIX white paper (CBOE web site)
- [2] Emanuel Derman, "Regimes of Volatility Some Observations on the Variation of S&P 500 Implied Volatilities", January 1999
- [3] Jim Gatheral "The Volatility Surface: A Practitioner's Guide", 2006
- [4] Jim Gatheral, Antoine Jacquier, "Arbitrage-free SVI volatility surfaces", 2013
- [5] Jim Gatheral "Valuation of Volatility Derivatives" 2005
- [6] Jim Gatheral "Consistant Modeling of SPX and VIX options", 2008
- [7] Kungliga Tekniska Hogskolan "The SVI implied volatility model and its calibration", 2014
- [8] Lorenzo Bergomi, "Smile Dynamics", 2004
- [9] John Wolberg "Expert Trading Systems", 2000